Overshoot effect in the Janssen granular column: A crucial test for granular mechanics

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We present experimental results on the mechanical behavior of static granular assemblies confined in a vertical column. Our measurements confirm the universal Janssen scaling for the stress saturation curve. We show consequently, in the context of isotropic elasticity, a relation between the Poisson ratio and the granular packing fraction. Moreover, using a systematic study of the overshoot effect created by a top mass equal to the saturation mass, we show behaviors that are reproduced qualitatively by isotropic elastic materials but of a spectacular amplitude in the case of a granular assembly. These experimental results are strong tests for any theory of granular matter.

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The mechanical status of granular matter is presently one of the most open and debated issues [1]. This state of matter exhibits many unusual mechanical and rheological properties such as stress-induced organization at the microscopic [2] or the mesoscopic [3] level. This issue sets fundamental questions relevant to the understanding of many other systems exhibiting jamming such as dense colloids or more generally soft glassy materials [4,5]. For practical applications, the quasistatic rheology of granular assemblies is described using a phenomenological approach, based on an elastoplastic modeling of stress-strain relations [6]. So far, there is no consensus on how to express correctly the macroscopic constitutive relations solely out of microscopic considerations and under various boundary conditions or loading histories. This very basic issue was illustrated in a recent debate on how to understand the stress distribution below a sand pile and especially how to account for the dependence on preparation protocols [7]. A new mechanical approach was proposed based on the concept of "fragile matter" [5] and force chain propagation modeling [8]. But recent experiments have evidenced results more consistent with the traditional framework of general elasticity [9].

In this paper, we obtain interesting and nonintuitive results on stress measurements at the bottom of a granular assembly confined in a rigid cylinder. Two kinds of experiments were performed. First, the mass at the bottom of the column is measured as a function of the granular material filling mass. Second, similar measurements are taken with an overweight on the top of the granular material. Similar experiments were previously made by Vanel *et al.* [10] who showed that contrary to previous experimental reports, one could obtain quite reproducible data, provided that there was a good control of the packing fraction homogeneity and a mobilization of all the friction forces at the walls in the upward direction. The existence of an overshoot effect was clearly demonstrated then, but neither interpretation nor systematic measurement was undertaken. Note that this over-

shoot effect is also an outcome of the "OSL theory" [8] based on a vault propagation modeling, but an important feature of the theory was not evidenced experimentally [10]: the existence of stress oscillations. In the present paper, we use the same experimental setup as in Ref. [10], but with a noticeable change in the measurement procedure which we claim is a decisive improvement for data interpretation.

The grains are dry, noncohesive, and slightly polydisperse (10%) glass beads of diameter $d=1.5$ mm piled into a vertical steel cylinder. A brass overweight can be added on top of the granular material. Three different column radii are used: $R=19$ mm, 28 mm, and 40 mm. The static coefficient of friction μ_s between the glass beads and the steel walls is measured using the sliding angle of a three-bead tripod. We found $\mu_s=0.25\pm0.02$. Two other steel cylinders of radius $R=19$ mm are used: a rough one ($\mu_s=0.285\pm0.02$) and a smoother one ($\mu_s=0.22\pm0.02$). The column is closed at the bottom by a movable piston (the diameter mismatch is 0.5 mm). A force probe of stiffness $k=40\,000$ N m^{-1} is located under the piston and can be moved at constant driving velocity V via a stepping motor (see inset in Fig. 1).

The experiments were performed on three different pilings: a dense one ($64.5\pm0.5\%$) obtained by rain filling, a loose one ($59.0\pm0.5\%$) obtained by using an inner cylinder which is removed slowly after filling, and a medium one ($62.0\pm0.5\%$) obtained by vibrating a loose packing.

For *each data point*, the same procedure is repeated: first, the column is filled with a mass of beads M_{fill} and prepared according to the desired average packing fraction ν. Then, the vertical translational stage is moved down slowly by a step motor (resolution of 100 nm/step) and the mass on the gauge is monitored. From the mass vs vertical descent data, the apparent mass M_a at the bottom of the column is extracted. We use a very slow descent of the force probe at a velocity $V_0=1.5$ μm s^{-1}, more than ten times smaller than the one used by Vanel *et al.* [10]. Then, a quasistatic situation is attained where all dynamical oscillations are suppressed. In the first stage of the descent, the granular material does not move (see inset in Fig. 1). The stress probe yields a value: $m(t)=m_0-(k/g)V_0t$, where m_0 is the apparent mass at time $t=0$ just after the end of the filling procedure (note that the value of m_0 is highly irreproducible from one filling

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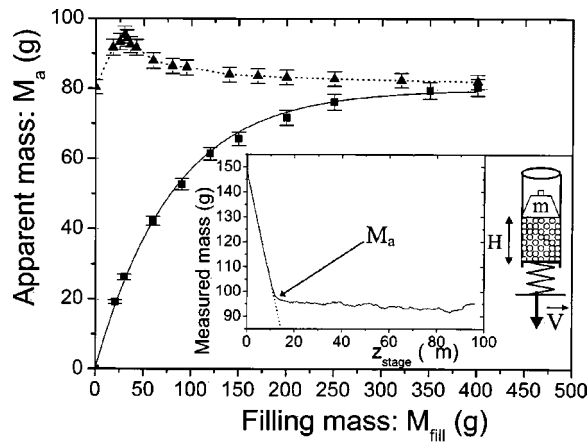
FIG. 1. Apparent mass M_a vs filling mass M_{fill} for a loose packing in the medium-rough-38-mm-diameter column without an overweight (squares) and with a 80.5 g overweight (triangles); the full line is the prediction for an elastic medium of Poisson coefficient $\nu_p = 0.46$. Inset: measured mass vs translation stage displacement Δz_{stage} for a loose packing in the smooth 38-mm-diameter column; the arrow indicates the measure point; the dotted line corresponds to the linear spring relaxation (slope $-k/g$). Sketch of the experimental display.

to the other). When the friction at the walls reaches the Coulomb threshold, the material starts to slip and then gently stabilizes at the value $V(t) \simeq V_0$. Then, the apparent mass may decrease for a piston descent of another 50 μm or more (the precise value depends on the column size) before it reaches a stationary state. This latter evolution could be due to mechanical restructuring of the piling caused by the slippage at the boundaries. This evolution to a plateau limit should not be confused with a dilatancy effect, which would occur much later, for displacements of about a grain size and also would lead to an increase of the apparent mass [10]. We choose to make the apparent mass (M_a) measurements, just at the end of the linear relaxation regime (see arrow in the inset of Fig. 1). We verified, using a displacement probe at the top part of the pile, that motion of the bottom and the top starts simultaneously. Therefore, this procedure should ensure (i) that the static piling is at the threshold of slippage *everywhere* at the walls and (ii) that we measure the mechanical properties of the initial piling in the absence of mechanical restructuring (which may be different with or without an overweight). These are the major differences from the procedure of Vanel *et al.* [10] who drive the granular material down, then stop the movement, and extract the apparent mass at rest after relaxation (i.e., the material is not at the threshold of slippage at the walls, though it may be near this state). In cylindrical coordinates with origin at the top surface and the cylinder axis being the z axis, the relation, at the slipping onset, between the shear stress σ_{rz} and the horizontal stress σ_{rr} at the walls is $\sigma_{rz}(r=R,z) = \mu_s \sigma_{rr}(r=R,z)$ (μ_s is the Coulomb static friction coefficient between the grains and the wall).

The typical results obtained when the filling mass M_{fill} is varied are shown on Fig. 1. The apparent mass M_a saturates exponentially with M_{fill}. When an overweight equal to the

saturation mass M_{sat} is added on top of the granular material, M_a increases with M_{fill}, up to a maximum M_{max}, and then decreases slowly towards the saturation mass M_{sat}. We also verified that if we add and remove slowly the overweight just after filling, and then apply our measurement procedure, we get an apparent mass corresponding to the original Janssen saturation curve. It means that adding an overweight does not modify significantly the initial packing structure.

The simple model which captures the physics of this saturation phenomenon was provided in 1895 by Janssen [11]. This model is based on the equilibrium of a granular slice taken at the onset of sliding and predicts a relation between the filling mass M_{fill} and the apparent mass at the bottom M_a of the form

$$M_a = M_{sat}\left[1 - \exp\left(-\frac{M_{fill}}{M_{sat}}\right)\right] \quad \text{with} \quad M_{sat} = \frac{\rho \pi R^3}{2K\mu_s}, \quad (1)$$

where ρ is the mass density of the granular material and K is the Janssen parameter rendering the average horizontal redirection of vertical stresses $\sigma_{rr} = K\sigma_{zz}$. From a mechanical point of view, a major simplification of this model comes from the assumption that the redirection parameter K would stay constant along the vertical direction. But on the other hand, it provides a clear and simple physical explanation for the existence of an effective screening length $\lambda = R/2K\mu_s$ above which the mass weighted at the bottom saturates. Many saturation profiles were measured for various packing fractions, columns sizes, and friction coefficients. Janssen's model may look very elementary but when the apparent mass rescaled by the saturation mass is plotted as a function of the filling mass also rescaled by the saturation mass, we obtain a *universal rescaling* of all data on a curve which is precisely the one predicted by Janssen: $M_a/M_{sat} = f(M_{fill}/M_{sat})$, with $f(x) = 1 - \exp(-x)$ (see Fig. 2). We recall that each data point corresponds to an independent experiment. The result is a priori so surprising that we will try next to understand it in reference to a simple elastic modeling of the granular column. The differences in M_{sat} measured for different densities means that the effective Janssen coefficient K increases with the average packing fraction (Fig. 2 inset). In other words, the efficiency of vertical stresses redirection is better for higher packing fractions and it is a very sensitive result ($\Delta K/K \simeq 5\Delta\nu/\nu$). Note, however, that there may be more structural differences between the pilings other than the packing fraction. We also obtain a Janssen coefficient K independent of radius R (i.e., M_{sat} scales with R^3) and the friction coefficient μ_s (inset in Fig. 2). These results are to the best of our knowledge the *first* experimental validation of this 1895 model.

For experiments with an overweight on the top, we also rescale the data for different column sizes [Fig. 3(a)] and also find a universal curve $g(x)$ such that $M_a/M_{sat} = g(M_{fill}/M_{sat})$, which means that the data scale with R^3 as M_{sat} does. Note that this overshoot goes beyond the possibilities of Janssen's model which would predict a flat curve: $M_a/M_{sat} = 1$. But, as we change density or friction, we find no rescaling of the data. In the inset of Fig. 3(b), we display
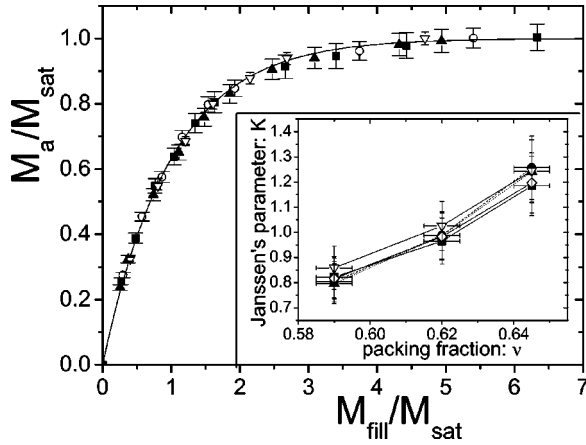
FIG. 2. Apparent mass M_a vs filling mass M_{fill}, rescaled by the saturation mass M_{sat}, for a loose packing in the smooth 38-mm-diameter column (triangles), for a dense packing in the rough (open circles) and smooth (squares) 38-mm-diameter columns, and for a dense packing in the 80-mm-diameter column (open down triangles). The results are compared to Janssen's model prediction (line). Inset: Janssen's parameter K extracted from experiments carried out in medium-rough columns of three diameters: 38 mm (squares), 56 mm (circles), 80 mm (triangles), and in a smooth (open down triangles) and rough (open diamonds) 38-mm-diameter column vs packing fraction.

the maximum overshoot value M_{max} rescaled by the saturation mass M_{sat} and show that its value increases with packing fraction and friction at the walls. Thus, we can reformulate this result in a very surprising and nonintuitive way: *the more the weight of the grains is screened by the walls, the less the weight of the overload is screened*. Note that a R^3 scaling for the apparent masses (valid with and without overweight) rules out theoretical predictions [12] based on the q model [13]. Furthermore, a fit to our data using the two-parameter OSL model shows inconsistency due to the generic presence of strong stress oscillations in this model [10].

We now study in detail the predictions of isotropic homogeneous elasticity. Here, we focus on the predictions related to our experimental results; many details will be given elsewhere [14]. We consider an elastic medium, of Young modulus E and Poisson coefficient ν_p, in a column of radius R. Stresses and displacements can be calculated in the limit of high depths z under the assumption that they should be independent of z. The boundary conditions we impose are the Coulomb condition everywhere at the walls $[\sigma_{rz}(r=R) = \mu_s \sigma_{rr}(r=R)]$, and infinitely rigid walls i.e., $u_r(r=R) = 0$. Using the stress-strain relation $E\epsilon_{ij} = (1+\nu_p)\sigma_{ij} - \nu_p \delta_{ij}\sigma_{kk}$ and internal equilibrium relation $\partial_i \sigma_{ij} = -\rho g_j$, we find $\sigma_{rr} = \sigma_{\theta\theta} = [\nu_p/(1-\nu_p)]\sigma_{zz}$ with $\sigma_{zz}^{sat}(r,z) = -\rho g R/2\mu_s$. The asymptotic displacements are

$$u_z(r,z) = -\frac{1+\nu_p}{2E}\rho g r^2 - \frac{1-\nu_p-2\nu_p^2}{2\mu_s \nu_p E}\rho g R z + u_0$$

and $u_r(r,z) = u_\theta(r,z) = 0$. Thus, we obtain a Janssenlike redirection phenomenon due to a Poisson ratio effect with a local Janssen parameter $K_{el} = \sigma_{rr}(r,z)/\sigma_{zz}(r,z)$, being for large depths

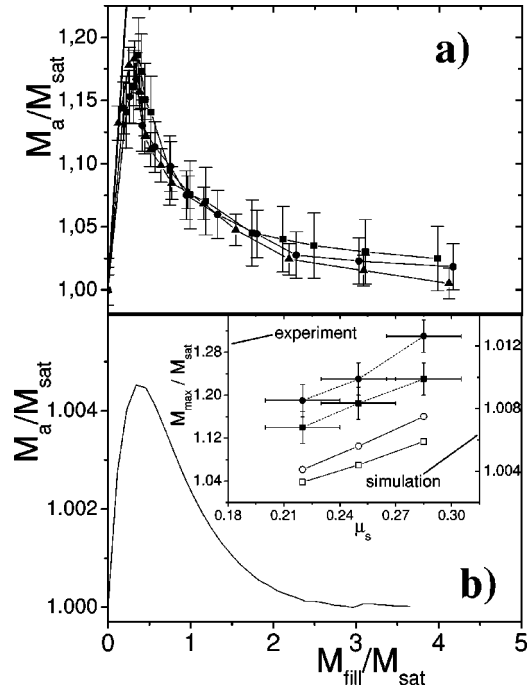FIG. 3. (a) Apparent mass M_a vs filling mass M_{fill}, rescaled by the saturation mass M_{sat}, for loose packings in medium-rough columns of three diameters [38 mm (squares), 56 mm (circles), and 80 mm (triangles)] with an overweight equal to M_{sat}; the full line is the hydrostatic curve. (b) Simulation of the experiment of (a) for an elastic medium characterized by the same saturation mass (Poisson coefficient $\nu_p = 0.46$) and the same friction at the walls ($\mu_s = 0.25$). Inset: maximum mass M_{max} rescaled by saturation mass M_{sat} vs static coefficient of friction at the walls μ_s, in experiments made on loose (squares) and dense (circles) packings in 38-mm-diameter columns, and in simulations for elastic media of Poisson coefficients $\nu_p = 0.45$ (open squares) and $\nu_p = 0.49$ (open circles); the left vertical scale is used for the experimental data and the right vertical scale is used for the simulation data.

$$K_{el} = \nu_p/(1-\nu_p). \tag{2}$$

Thus, in the context of isotropic elasticity, the dependence of K on packing fraction can be interpreted as an increase of the Poisson ratio ν_p with packing fraction. Note that the stress profile in an elastic column was derived by Evesque and de Gennes [15] for $\lambda \gg R$ (not realized in our experiment) and for different boundary conditions (partial mobilization the friction forces along the walls) and they obtained a similar asymptotic limit.

In order to get the whole stress saturation curve, finite element numerical simulations [16] were performed. The column is modeled as an isotropic elastic medium. We vary the friction at the walls μ_s, the Young modulus E, and the Poisson coefficient ν_p. We imposed a rigid, either perfectly stick or perfectly slip bottom. We found no appreciable difference between these two previous cases. The condition $\sigma_{rz} = \mu_s \sigma_{rr}$ is imposed everywhere at the walls. The cylinder is modeled as a steel elastic medium. As long as the Young modulus E of the elastic medium is less than 500 MPa, which is usually the case for granular media, we find no

dependence of the results on E. We verified that in all the simulations performed, there is no traction in the elastic medium, so that this can actually be a model for a granular material. We observed indeed that for low friction coefficients, i.e., $\mu_s < 0.5$, the stress curve at the bottom of the column could not be separated from Janssen's universal curve (see Fig. 1: the data are well fitted by the elastic curve). Note that the presence of a bottom implies that the effective Janssen parameter K_{el}^{eff} extracted from Janssen's scaling is higher than K_{el}. We have in fact $K_{el}^{eff} = a K_{el} + b$ with $a = 1.029 \pm 0.002$ and $b = -0.008 \pm 0.001$. If we use the correspondence with the effective Janssen constants obtained experimentally, this would yield an empirical relation between Poisson ratio ν_p and packing fraction ν as $\nu_p \simeq 2.3(\nu - 0.41)$ with a precision of 5%. Note that the largest packing fraction $\nu = 0.645 \pm 0.005$ would give a Poisson ratio $\nu_p = 0.54 \pm 0.03$ marginally larger than the limit value of 1/2. Note that (i) nothing really ensures that our piling preparation is strictly isotropic and (ii) in spite of the careful procedure, we are never absolutely sure that all the friction forces at the wall are actually fully mobilized upwards. We also performed simulations with an overweight (which is taken perfectly stick or slip with no appreciable difference), using brass mechanical parameters. We found that the resulting curve is very similar to the experimental one, but the experimentally observed maxima are consistently 30 *to* 40 *times larger* [see Fig. 3(b)]. This is what we would call in this context a *giant overshoot effect*. We find from numerical computations the same qualitative phenomenology as in the experiment, i.e., the computed values of the overshoot M_{max} rescaled by the saturation mass M_{sat}, increase both with the friction at the walls and with the Poisson coefficient (i.e., with the effective Janssen parameter). Basically, we recover in the elastic case, the same paradox as the experimental situation. We now understand it as a consequence of the boundary condition imposed experimentally by the overweight, i.e., an almost constant displacement on the surface: $u_z(r) = u_0$. When ν_p or μ_s is decreased to zero (i.e., when M_{sat} increases), the parabolic part of the asymptotic vertical displacement becomes negligible and, therefore, the displacement on the surface is close to the asymptotic value. This leads to a flat pressure profile.

Understanding the amplitude of this overshoot effect is still an open question but we propose here a qualitative vision based on stress-induced anisotropy. Small mechanical restructuring due to displacements close to the overweight at the top could be capable of producing local anisotropy in a way such that the vertical direction would become stiffer due to the preferential increase in the number of granular contacts. A consequence of this anisotropy is that the overshoot amplitude can be increased. This is what we verified numerically using a model of orthotropic elasticity [16] with the stiff axis taken in the vertical direction. But importantly, we also need to assume that this restructuring effect has a finite spatial extent since for large column heights the original saturation limit is recovered.

In summary, we performed a series of precise experiments on a granular material confined in cylindrical columns. We obtained the universal scaling predicted by Janssen in 1895 for the stress saturation curve when the filling mass is increased. We show that the redirection parameter (the effective Janssen constant) is an increasing function of the average packing fraction. By comparison with isotropic elastic theory, we provide a one-to-one correspondence between the Poisson ratio and the granular packing fraction. We study systematically the overshoot effect, i.e., when a mass equal to the saturation mass is added to the top of the packing we have a nonmonotonic variation of the apparent mass as a function of the filling mass. This effect is reproduced qualitatively by isotropic elasticity but of a scale 30 to 40 times smaller. We suggest a qualitative interpretation of this effect based on stress-induced anisotropy. Therefore, along these lines this overshoot effect can be seen as a strong test for any theoretical model describing the statics of granular assemblies under various loading histories and boundary conditions.

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